A STUDY ON THE OPTIMIZATION ALGORITHMS FOR DETERMINING OPEN-PIT AND UNDERGROUND MINING LIMITS


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ABSTRACT

Determination of Mining Limits (ML) for each of Open-Pit (OP) and Underground (Ug) is one of the most significant parts impacts the economic aspects. Optimization process for the mentioned objective in both of OP and Ug mining was done on the economic block model. In this paper, the applied Optimization Algorithms (OAs) for ML determination extensively were studied. Because of the major difference between the shape and form of mining limits in two main exploitation methods, implementation of the algorithms in searching and founding the mining limits for OP and Ug is also different.

Keywords: Optimization algorithm, open-pit, underground, mining limit, determination

INTRODUCTION

In the last 40 years or so that modern computers were invented various form of techniques to judge and select one choice out of the many options of mine planning were used. Optimization has now become such a taboo word in mine planning that managers and financiers almost routinely ask of any plans submitted for approval, “Have they been optimized?” [1]. Underground operations and its mining limits (boundaries) are less amenable to optimization techniques. Optimization techniques frequently refer to economic criteria which require each flow projections. Although, implementation of mining limits OAs initially were due to pit limits optimization, but during the last years these algorithms were developed as the optimization tools for determination of underground mining limits. In mining, there are three principal kinds of models (ore body models, financial or economical models, and mine models) commonly employed in developing long term mine plans. Mine models are described as a form of established structures. Like underground mine will have shafts or inclines, outwardly and downwardly expanding stope or panels, drifts and cross-cut and funnels or chutes joining with the level drifts. The pit is usually represented by a series of levels (easily conceived as benches) and a set of perimeter, one on each level expanding upwards from one level to the next. In mining, financial or economical models are commonly utilized for planning, budgeting and economic evaluations. Up to now, the whole of presented mining limits OAs were based on the financial blocks model (with economical and grade values).

OPTIMIZATION CONCEPT

Optimization in term of mine planning is known as descriptive of a set of techniques that introduces analytical mathematical methods to arrive at an option out of multiple
choice—thus bringing it into planning activities. The techniques embody a process of three stages [2].

1. The creation of a mathematical model of the activity.
2. The adoption of criterion.
3. The development of an algorithm.

CLASSIFICATION OF OAs FOR MINING LIMITS DETERMINATION

In 1978 Kim described the various optimization techniques. His classification is included rigorous (in the sense that the algorithm can be proved to find optimum solution) and heuristic [3]. According to the two mentioned optimization techniques (rigorous and heuristic), mining limits OAs based on economical block models are classified in two main groups named open pit and underground. In this case, the most significant algorithms which are studied in this paper for both of mining methods are summarized in two classes as figure 1.

![Figure 1: Classification of mining limits optimization algorithms based on economical block models](image)

INVESTIGATION OF PIT LIMITS OAs

A number of OAs to solve the basic open pit mining problem especially for pit limit determination have been developed over the years. These algorithms include rigorous and heuristic approaches. Almost all optimal open pit design algorithms, with the exception of elementary methods that are used for some stratiform deposits, are applied to a regular, fixed, three dimensional block model of orebody. The orebody is subdivided into regular blocks and a value is estimated for each block. This value is almost always the net (undiscounted) revenue that would be obtained by mining and treating the block and selling
its contents. Some methods such as parameterization (Parametric Analysis), used grade values in the block model [4]. The entire mentioned pit limit OAs are discussed in the next sections.

**PIT LIMITS OAs BASED ON THE RIGOROUS LOGIC**

**A) DYNAMIC PROGRAMMING (DP)**

The optimum final pit limit is that combination of mined blocks which maximizes profit within practical constrains, such as the stability of the pit slopes and ease of access for machinery and trucks from one level to the next [5]. Two dimensional Lerchs and Grossmann algorithm (1965) was the first pit limit optimization technique based on dynamic programming (DP) algorithm [6]. In 1971, Johnson and Sharp developed a three dimensional DP algorithm which is a direct extension of Lerchs and Grossmann. There was basically two-step procedure, first repeating LG along all cross-sections and then applying it to a longitudinal section [7]. This satisfies constraints on the individual cross-sections well, but not in the longitudinal direction, and so some smoothing is still required. In 1982, a new three dimensional DP algorithm which conducts a search, subject to the geometrical constraints, through a fully three dimensional array of blocks was introduced by Koenigsberg [8]. Wilke et al, (1984) were proposed a modification of Koenigsberg, s formulation that will always yield a non-degenerate solution [9]. However, to achieve reasonable computing times and storage requires a number of modifications that render the algorithm sub-optimal [10].

**B) GRAPH THEORY (GT)**

Lerchs and Grossmann (1965) were represented a new pit limit OA based on rigorous reasons named Graph Theory (GT) which makes use of property of a block model of an open cut mine that it can be modeled as a weighted-directed graph in which the vertices represent blocks and the arcs represent mining restrictions on other blocks. Early version of the GT algorithm, as described by Zhao and Kim (1992) operates by applying a set of transformations to the graph’s tree structures until there are no waste blocks preventing the removal of desirable ore blocks [11], [12]. The GT algorithm is able to ensure the true optimality of pit design.

**C) PARAMETRIC ANALYSIS (PA)**

One radically different approach to pit optimization is to parameterize the pit design as a function of a number of variables. This approach was pioneered by Matheron [13] and have been described in a number of places [14], [15], [16], [17]. This algorithm is to divide the problem into two separate parts- technical and economical- and assume that the only pit design of any interest is that which maximizes the quantity of metal. In the latest improvement, Whittle (1988) introduced an economic parameter as the ratio of metal price to the mining cost and developed the Whittle 4-D software, which generates incremental pits by increasing this economic parameter [18].
D) STOCHASTIC PROCESS (SP)
Stochastic Process (SP) algorithm as a pit limit optimization technique was initially used and introduced by Jalali et al (2006) on the principles of SP and benefits from a mathematical support based on the probability theory [19].

PIT LIMIT OAs BASED ON THE HEURISTIC LOGIC
Up till the early 1980s, heuristic algorithms were widely used in the mining industry because they execute faster and are conceptually simpler than other optimizing algorithms. The algorithms based on the heuristic techniques are not guaranteed to deliver optimal solutions as well as optimization approaches [20].

A) MOVING CONE (MC)
Moving Cone (MC) algorithm which described by Pana (1965) is to search for cones in which the total weight of all the blocks in the cone is positive [21]. These cones are added to the already generated pit. The heuristic idea lies in the assumption that every cone in the optimal pit is profitable, whereas in fact an optimal pit may consist of a collection of cones share negative value blocks and have total weight which is positive. This algorithm is very simple in concept, easy to program and reaches a solution in quite a short time. However, the variants that have been published to date do not always yield a true optimum, and this can usually be demonstrated by means of simple counter-examples [22], [23].

B) NETWORK FLOW ANALYSIS (NFA)
In 1968, the use of Network Flow Analysis (NFA) to solve the ultimate pit limit determination problem for an open cut mine was first proposed by Johnson [24]. Picard (1976) developed a formal proof for the concept [25]. Giannini et al (1991) developed the software package PITOPTIM, which uses maximum flow algorithm for computing the optimal pit contour [26]. Yegulalp and Arias (1992) and Yegulalp et al (1993) conducted computational experiments to compare the performance of a variant of the push-relabel maximum flow algorithm, the excess-scaling algorithm of Ahuja and Orlin (1989), and Lerchs and Grossmann algorithm [27], [28], [29].

C) LINEAR PROGRAMMING (LP)
Meyer (1969) was introduced Linear Programming (LP) technique to optimize pit limit. However, the most significant disadvantage of the approach was its excessive need for computer memory and computing time that made it impractical [30]. In 1994, Huttagosol and Cameron formulated ultimate pit limit problem as a large scale transportation problem with an LP solution [31]. The adapted simplex algorithm of LP is used to solve the dual system. The required computing time and memory are the main constraints of the method especially for a large scale problem.

D) KOROBOV (K)
The Korobov algorithm (1974) is a cone-based algorithm that allocates the values from positive blocks against the negative or zero blocks that are contained in the extraction cones
of the positive blocks. A flow chart for the algorithm has been presented else where [32], [33]. A major improvement to the Korobov algorithm was suggested by Dowd and Onur (1992), in which they propose joint testing of cones before it decided whether or not to remove a cone [34].

E) GENETIC ALGORITHM (GA)
Firstly, Denby and Scholfield (1994) employed Genetic Algorithm (GA) as a optimization tool in open pit design [35].This technique was established on the basis of a self-training GA and concurrently supplies a combined pit limit and extraction schedule that intends to maximize the net present value (NPV) of the production.

F) ARTIFICIAL INTELLIGENCE (AI)
Achireko and Frimpong (1996) applied Artificial Neural Networks (ANN)s in combination with “Conditional Simulation” technique to optimize the pit limits [36]. They referred the random field properties connected to the ore grades and reserves, which are ignored in preceding algorithms and displayed them using the “Modified Conditional Simulation” (MCS) based on the best linear unbiased estimation and local average subdivision techniques. ANNs are used to classify the block into classes based on their conditioned economic values.

INVESTIGATION OF UNDERGROUND LIMITS OAs
There are few algorithms available to optimize underground stope boundaries, such as: Dynamic Programming (DP), Branch and Bound (B&B), Floating Stope (FS), Octree Division (OD), Downstream Geostatistical (DG), Maximum Value Neighbourhood (MVN), Probable Stope (PS). OD and DG techniques are implemented on the grade block model of ore body. But the other mentioned algorithms were performed on the economic block models. In this study, the algorithms based on economic block model were investigated.

UNDERGROUND LIMITS OAs BASED ON THE RIGOROUS LOGIC
A) DYNAMIC PROGRAMMING (DP)
Initially, the DP algorithm was used by Riddle (1977) to optimize stope layout of block caving method [37]. This method was established due to modifying the Johnson and Sharp (1971) approach [7]. The DP algorithm by Riddle is a multi-section two dimensional solution for three dimensional problems. It means the approach is provided an optimum stope in two-section, but it is failed to determine the actual optimal stope in three dimensions. It is notable that this method is limited to the block caving mines and can not be able in optimizing the layout of other underground stopping methods.

B) BRANCH AND BOUND (B&B)
Ovanic and young (1995) used a separable programming with Branch and Bound (B&B) technique for economic optimization of stope boundary. An optimal economic stoping boundary was developed by optimizing the starting and ending locations for mining
within each row of blocks. To determine these locations, two piecewise linear, cumulative functions are used for each row. The first function sums block values along the row for inclusion within the stope boundary, while the second function sums block values for exclusion. In this algorithm the stope boundary model is optimized by a mixed integer approach known as "special ordered sets" [38],[39].

C) PROBABLE STOPE (PS)
Jalali and Ataee-pour (2004) established a technique (named Probable Stope) based on Riddle DP algorithm (1977) to optimize the stope limits of the mining methods which are feasible for vein deposits [40]. There is a main difference between PS technique and the others. This algorithm is implemented basis on a particular economic block model including the constraints of stope dimensions. Therefore, the most significant constraints within the objective function are eliminated. It is resulted the algorithm be simple in concept, easy to program and reaches a solution quickly.

UNDERGROUND LIMITS OAs BASED ON THE HEURISTIC LOGIC

A) FLOATING STOPE (FS)
Floatin Stope (FS) was implemented on a fixed economic block model of the orebody. The FS is the tool developed by Datamine to define optimal limit for mineable ore or stope envelope which can be economically extracted by underground stoping methods. The general concept of the FS method was established by Alford (1995) [41].

C) MAXIMUM VALUE NEIGHBOURHOOD (MVN)
Ataee-pour (1997) proposed the MVN algorithm to optimize stope boundaries using a three-dimensional fixed economic block model to locate the best neighbourhood of a block, which guarantees the mine geometry constraints. The neighbourhood concept is based on the number of mining blocks equivalent to the minimum step size. Since several neighbourhoods are available for each block, the one that provides the maximum net value is located for inclusion in the final stope. It can apply to any underground mining method, although it does not guarantee the true ‘optimum’ stope layout [42].

CONCLUSIONS
There are main and basic differences between the specifications of an open-pit and those of a stope, which influence mining technology to be employed. The actual optimal solution is guaranteed for the pit limit optimization and several computer packages are available. However, only few algorithms have been developed for optimizing final stope limits in underground mining. Up to now, the all presented mining limits OAs were based on the financial blocks model (with economical and grade values). These algorithms were classified as rigorous and heuristic. It is notable that the problem of true optimally has not been solved in the stope limit optimization yet except FS algorithm of Datamine.
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